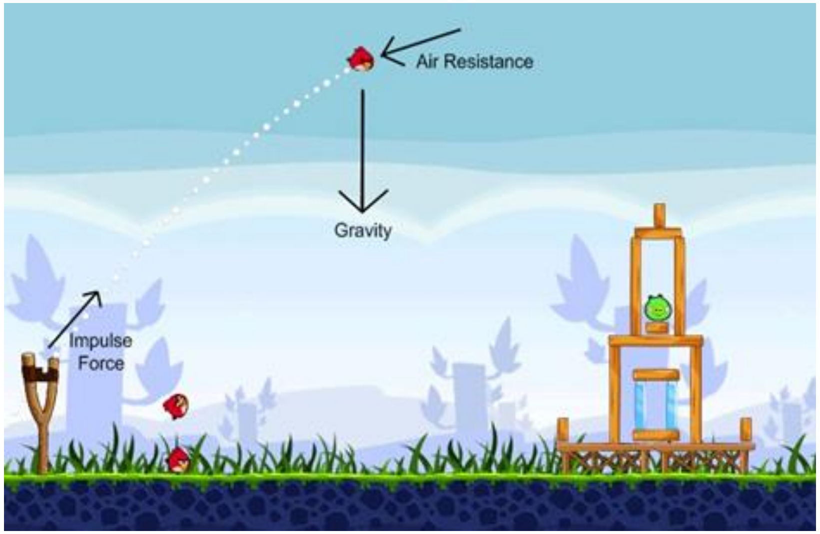
$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

Quantum
Lie algebras
and
Dissipation

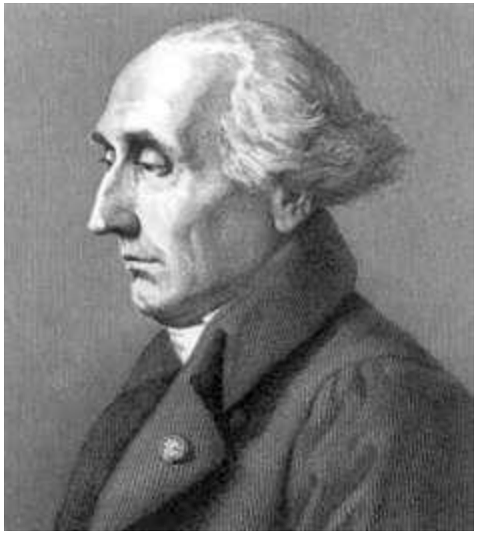
Alberto Acevedo

Classical mechanics

- $x(t)$
- $F(t,x) = m(t,x)a(t)$
- $a = \frac{d^2}{dt^2}x(t)$
- $m(t,x)\frac{d^2}{dt^2}x(t) - F(t,x) = 0$



Fancy!



- $T = Kinetic\ energy$
- $V = Potential\ energy$
- $L = T - V$
- $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$ $\quad Lagrangian$

- $H = T + V$ $\quad Hamiltonian$
- $\frac{dp}{dt} = -\frac{\partial H}{\partial x}$
- $\frac{dx}{dt} = -\frac{\partial H}{\partial p}$

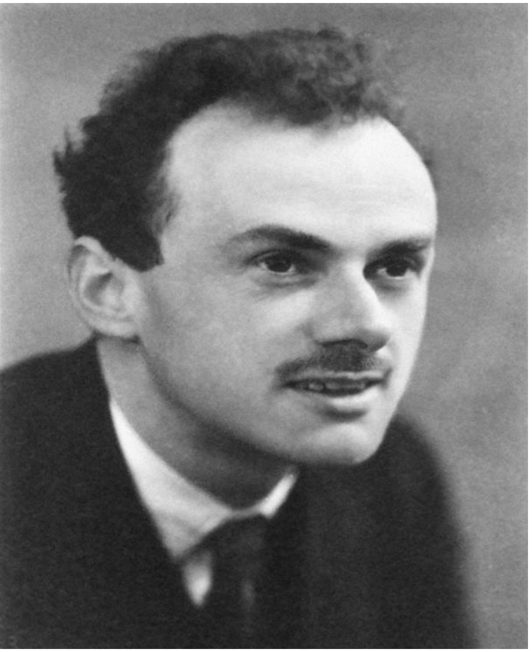
Quantum mechanics

- $x(t)$? ?
- NO!!!!!!!!
- $|\psi(t)\rangle$ Kets!

- $|\psi(t)\rangle = \alpha(t)|\text{Dead}\rangle + \beta(t)|\text{Alive}\rangle$
- $|\alpha(t)|^2$= Probability of dead kitty.
- $|\beta(t)|^2$= Probability of living kitty.

The time dependent Schrödinger equation.

Letting planks constant equal one.



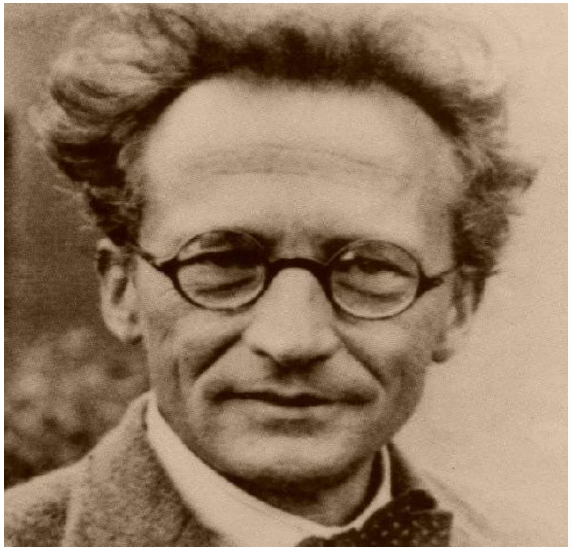
$$i\frac{d}{dt}|\psi(t)>= H(t)|\psi(t)>$$

$$|\psi(t)>= U(t,t_0)|\psi(t_0)>$$

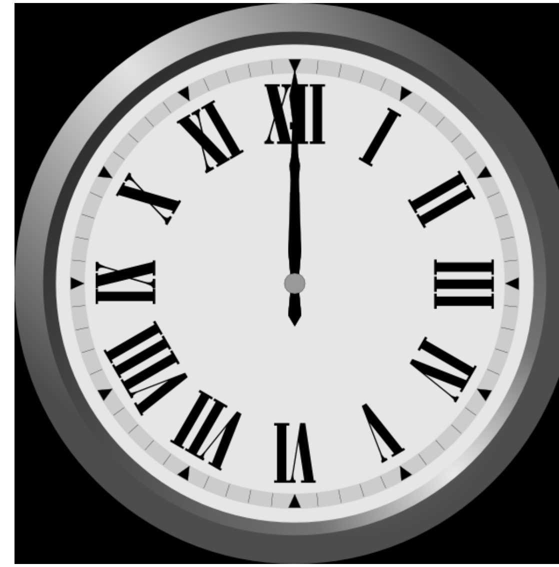
$$i\frac{d}{dt}U(t,t_0)|\psi(t_0)>= H(t)U(t,t_0)|\psi(t_0)>$$

$$i\frac{d}{dt}U(t,t_0) = H(t)U(t,t_0).$$

$$i\frac{d}{dt}U(t) = H(t)U(t),$$



Time Development



Case 1: H is time independent.

$$U(t) = e^{-iHt}$$

Case 2: H is time dependent and commutes with itself at different times.

$$U(t) = e^{-i\int_0^t H(t)dt}$$

Case 3: *H is time dependent!! Hard problem because operators do not commute in general.*

$$AB \neq BA$$

Lie Algebra method

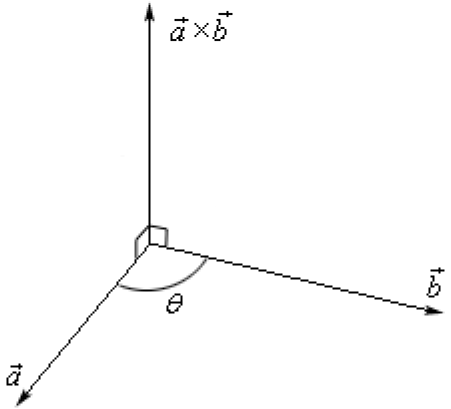
$$H(t) = \sum_{=0}^{n} f(t)_n A_n$$

$\{A_1, A_2,\ldots, A_m\}$ m>n

$[A,B]=AB-BA$

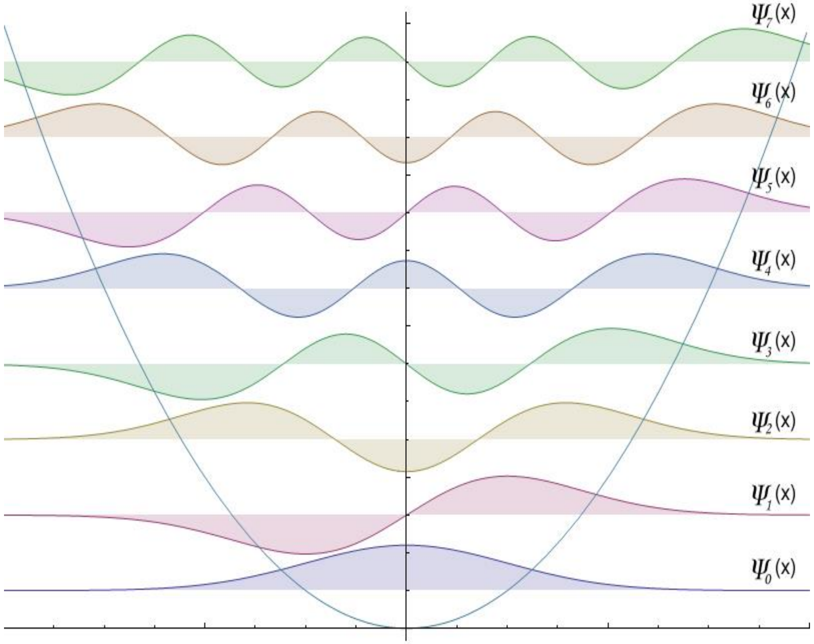
$$i(\frac{d}{dt}U)U^{-1} - H = 0$$

$$U(t) = e^{\alpha_1(t)A_1}\, e^{\alpha_2(t)A_2} \ldots e^{\alpha_n(t)A_n}$$



$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

SHO



$$H = \omega(N + \frac{1}{2}) \qquad \{N, \mathbb{1}\}$$

$$i\dot{U}(t)U(t)^{-1} - H = 0.$$

$$U(t) = e^{\alpha(t)N} e^{\beta(t)\mathbb{1}}$$

$$i\dot{U}U^{-1} = i\dot{\alpha}N + i\dot{\beta}\mathbb{1}$$

$$i\dot{U}U^{-1} - H = (i\dot{\alpha} - \omega)N + (i\dot{\beta} - \frac{1}{2}\omega)\mathbb{1} = 0$$

$$\begin{aligned} i\dot{\alpha} - \omega &= \quad \alpha(t) = -i\omega t \\ i\dot{\beta} - \frac{1}{2}\omega &= \quad \beta(t) = -\frac{i\omega t}{2} \end{aligned}$$

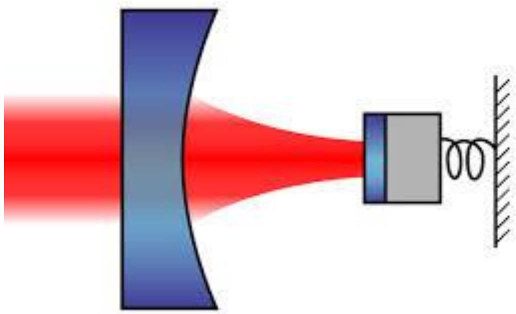
$$U = e^{-i\omega t N} e^{-\frac{i\omega t}{2}\mathbb{1}} = e^{-iHt}$$

Optomechanics

$$H = \omega_m b^\dagger b + \omega_c a^\dagger a - GN + f(t)(b^\dagger + b)$$

Basis= $\{N, b, b^\dagger, b^\dagger b, N^2, Nb^\dagger, Nb, \mathbb{1}\}$

$$U = e^{\alpha N^2} e^{\beta N b} e^{\delta b} e^{\zeta b^\dagger} e^{\eta N b^\dagger} e^{\xi N} e^{\sigma b^\dagger b} e^{\tau \mathbb{1}}$$



$$\begin{aligned}
i\dot{\alpha} + i\beta\dot{\eta} - i\beta\eta\dot{\sigma} &= 0 \\
G + i\dot{\beta} + i\beta\dot{\sigma} &= 0 \\
-f(t) + i\dot{\delta} + i\delta\dot{\sigma} &= 0 \\
-f(t) + i\dot{\zeta} + i\zeta\dot{\sigma} &= 0 \\
G + i\dot{\eta} - i\eta\dot{\sigma} &= 0 \\
-\omega_c + i\beta\dot{\zeta} + i\delta\dot{\eta} + i\dot{\xi} - i\beta\zeta\dot{\sigma} - i\delta\eta\dot{\sigma} &= 0 \\
-\omega_m + \dot{\sigma} &= 0 \\
i\delta\dot{\zeta} - i\delta\zeta\dot{\sigma} + i\dot{\tau} &= 0
\end{aligned}$$

Recall systems of equations

$$3x + 2y - 4z = 17$$
$$3x + 3y - 3z = 18$$
$$-3x + 3y - 4z = 1$$

Systems of differential equations

$$
\begin{aligned}
i\dot{\alpha} + i\beta\dot{\eta} - i\beta\eta\dot{\sigma} &= 0 \\
G + i\dot{\beta} + i\beta\dot{\sigma} &= 0 \\
-f(t) + i\dot{\delta} + i\delta\dot{\sigma} &= 0 \\
-f(t) + i\dot{\zeta} + i\zeta\dot{\sigma} &= 0 \\
G + i\dot{\eta} - i\eta\dot{\sigma} &= 0 \\
-\omega_c + i\beta\dot{\zeta} + i\delta\dot{\eta} + i\dot{\xi} - i\beta\zeta\dot{\sigma} - i\delta\eta\dot{\sigma} &= 0 \\
-\omega_m + \dot{\sigma} &= 0 \\
i\delta\dot{\zeta} - i\delta\zeta\dot{\sigma} + i\dot{\tau} &= 0
\end{aligned}
$$

Solution for the time evolution operator of optomechanics

$$\alpha = \frac{G^2(1 - e^{i\omega_m t} + i\omega_m)}{\omega_m^2}$$

$$\beta = \frac{-1 + e^{i\omega_m t}}{\omega_m}$$

$$\delta = -e^{i\omega_m t}(\int_1^0 -ie^{-i\omega_m t'} f(t')dt' - \int_1^t -ie^{-i\omega_m t'} f(t')dt')$$

$$\zeta = -e^{i\omega_m t}(\int_1^0 -ie^{i\omega_m t''''} f(t'''')dt'''' - \int_1^t -ie^{i\omega_m t''''} f(t'''')dt'''')$$

$$\eta = \frac{e-i\omega_m t(-1 + e^{i\omega_m t})G}{\omega_m}$$

$$\begin{aligned}
\xi \; = \; & \frac{1}{\omega_m}(-i\omega_c\omega_m - G\int_1^0 -ie^{-i\omega_m t'} f(t')dt' + \\
+ \; & e^{i\omega_m} G\int_1^0 -ie^{-i\omega_m t'} f(t')dt' - \omega_m \int_1^0 -\frac{iGf(t'')}{\omega_m}dt'' - \\
- \; & \omega_m \int_1^0 \frac{ie^{i\omega_m t''}Gf(t'')}{\omega_m}dt'' - \omega_m \int_1^0 -ie^{-i\omega_m t''}G\int_1^{t''} -ie^{-i\omega_m t'} f(t')dk(1)dt'' \\
& + \\
+ \; & \omega_m \int_1^t \frac{-1}{\omega_m i}(\omega_c\omega_m + Gf(t'') - e^{i\omega_m t''}Gf(t'') - \\
- \; & \omega_m e^{i\omega_m t''} G\int_1^0 -ie^{-i\omega_m k(1)} f(t')dt' + \\
+ \; & \omega_m e^{i\omega_m t''} G\int_1^{t''} -ie^{-i\omega_m k(1)} f(t')dt')dt'')
\end{aligned}$$

$$\sigma = -i\omega_m t$$

$$\begin{aligned}
\tau \; = \; & -\int_1^0 -e^{i\omega_m t'''} f(t''') \int_1^0 -ie^{-i\omega_m t'} f(t')dt'dt''' - \\
- \; & \int_1^0 ie^{i\omega t'''} f(t''') \int_1^{t'''} -ie^{-i\omega_m t'} f(t')dt'dt''' + \\
+ \; & \int_1^t ie^{i\omega_m t'''} f(t''')(-\int_1^0 -ie^{-i\omega k(1)} f(t')dt' + \\
+ \; & \int_i^{t'''} -ie^{-i\omega_m t'} f(t')dt')dt'''
\end{aligned}$$

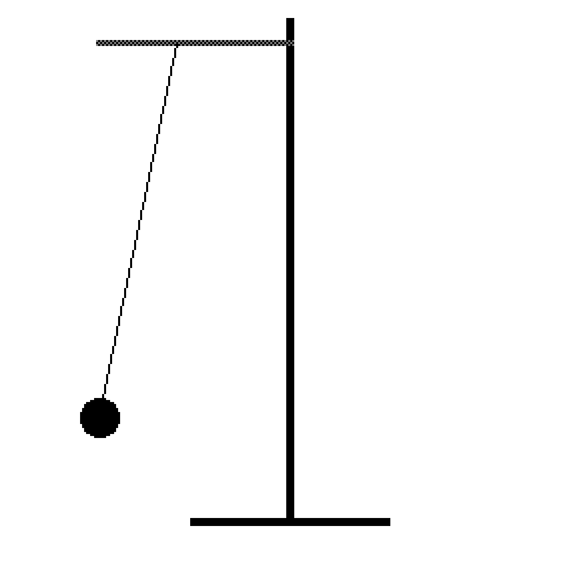
Is this manageable?

$$f(n,m) = m\frac{(n+2)(2n+1)}{2}$$

$$g(n,m) = m(2n+1) + \frac{n^2+3n}{2}$$

Not by Hand!

Dissipation in quantum mechanics.



$$H = N + \frac{1}{2}$$

$$H = H_0 \cosh(\frac{\gamma t}{2}) + \frac{1}{2}(a^\dagger a)\sinh(\frac{\gamma t}{2})$$

$$\{H_0, (a^\dagger)^2, a^2\}$$

$$-\frac{1}{2}\sinh(\frac{\gamma t}{2}) + i\dot{\alpha} - 2i\dot{\beta}\alpha + 4i\dot{\delta}\alpha^2 e^{-2\beta} = 0$$

$$i\dot{\delta}e^{-2\beta} - \frac{1}{2}\sinh(\frac{\gamma t}{2}) = 0$$

$$i\dot{\beta} - 4i\dot{\delta}\alpha e^{-2\beta} - \cosh(\frac{\gamma t}{2}) = 0.$$

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