Decoherence

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5/8/2020

Overview.

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- Closed systems. The Ammonia molecule.

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- Open Quantum Systems.

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- Decohere free subspaces.

The Ammonia Molecule, Example of Closed Quantum Systems.

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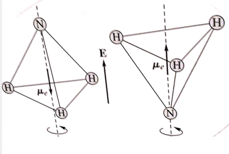

Hamiltonian

$$H \to \begin{pmatrix} E_0 & -\epsilon \\ -\epsilon & E_0 \end{pmatrix} = \begin{pmatrix} \langle 1|H|1\rangle & \langle 1|H|2\rangle \\ \langle 2|H|1\rangle & \langle 2|H|2\rangle \end{pmatrix}$$

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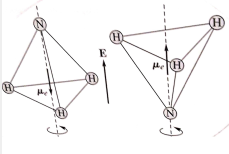

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Let us solve SE for $|\psi(0)\rangle = |1\rangle$

- $i\hbar\partial_t|\psi(t)\rangle = H|\psi(t)\rangle$
- $|\psi(t)\rangle = U(t)|\psi(0)\rangle$
- $U(t) = e^{\frac{-iHt}{\hbar}}$

Ammonia Molecule continued

Solution

$|\psi(t)\rangle = e^{\frac{-iE_0 t}{\hbar}}(\cos(\frac{\epsilon t}{\hbar})|1\rangle + i\sin(\frac{\epsilon t}{\hbar})|2\rangle)$, superposition principle at work.

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State Matrix

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)| \rightarrow \begin{pmatrix} \cos^2(\frac{\epsilon t}{\hbar}) & -i\cos(\frac{\epsilon t}{\hbar})\sin(\frac{\epsilon t}{\hbar}) \\ i\sin(\frac{\epsilon t}{\hbar})cos(\frac{\epsilon t}{\hbar}) & \sin^2(\frac{\epsilon t}{\hbar}) \end{pmatrix}$$

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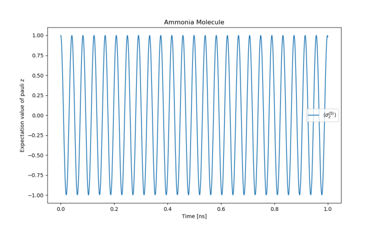
Time evolution of $Tr[\rho(t)\sigma_z]$



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Coherences in closed systems

For the Ammonia molecule, $|\rho_{12}(t)| = \rho_{21}(t) = \cos(\frac{\epsilon t}{\hbar}) \sin(\frac{\epsilon t}{\hbar})$. Note the periodic behavior. Closed systems have periodic coherences.

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Quantum computation

Quantum coherence is a vital cornerstone to the theory of quantum computation and quantum information. Quantum information is stored within quantum states and the superpostion principle is exploited in order to boost computational speed.

Open quantum systems

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- Dynamics provided by Schrödinger's equation.
  $i\hbar\partial_t|\psi_{SE}(t)\rangle = H|\psi_{SE}(t)\rangle$ where $H = H_S + H_E + H_I$, a *Hermitian* operator in $\mathscr{B}(\mathscr{H}_{SE})$.

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- $|\psi_{SE}(t)\rangle = e^{-\frac{it}{\hbar}H}$. Just like before.
  We can attain the reduced dynamics by partial tracing over the degrees of freedom pertaining to the environment. i.e.

$$\rho_S(t) := Tr_E\{|\psi_{SE}(t)\rangle\langle\psi_{SE}(t)|\}$$

Partial Trace

Definition

$Tr_E\{\} : T(\mathscr{H}_S \otimes \mathscr{H}_E) \to T(\mathscr{H}_S)$

$$Tr_E\{|\psi_{SE}(t)\rangle\langle\psi_{SE}(t)|\} := \sum_k \langle\phi_k|\psi_{SE}(t)\rangle\langle\psi_{SE}(t)|\phi_k\rangle,$$

where $\{|\phi_k\rangle\}_k$ is an ONB for $\mathscr{H}_E$.

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$$A_S \to A_S \otimes I_E,$$

$$\langle A_S \otimes I_E\rangle = Tr\{\rho_{SE}(A_S \otimes I_E)\}.$$

But it can be shown that

$$Tr\{\rho_{SE}(A_S \otimes I_E)\} = Tr\{\rho_S A_S\}!!!!.$$

Product state

Product state as an initial state

Assume our initial state to be in a product state.
$\rho_{SE}(0) = \rho_S(0) \otimes \rho_E(0) \in \mathscr{D}(\mathscr{H}_{SE}) :=$ Space of trace class operators over $\mathscr{H}_{SE}$ with trace one.

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Diagonalizing the environmental component

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Non-Unitary Time Evolution

$\rho_S(t) = Tr_E\{U(t)(\rho_{SE}(0))U^\dagger(t)\} = \sum_{ij} p_i \langle E_j|U(t)|E_i\rangle \rho_S(0) \langle E_i|U^\dagger(t)|E_j\rangle.$

We short hand this evolution as $\nu_t \rho_S(0) = \rho_S(t)$. (Dynamical map).

Krauss operators and completely positive maps.

Krauss operators

The operators $\langle E_j|U(t)|E_i\rangle \in \mathscr{B}(\mathscr{H}_S)$ are referred to as krauss operators. These operators characterize the dynamical map seen in the previous slide.

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Complete positivity

$\nu_t \otimes I_n$ required to be positive for all n. Without the latter we could end up mapping from positive operators to operators which are not (Negative probabilities).

Convex Linearity, evolving mixed states.

$$\nu_t\{\lambda\rho_{S_1}(0) + (1-\lambda)\rho_{S_2}(0)\} = \lambda\nu_t\rho_{S_1}(0) + (1-\lambda)\nu_t\rho_{S_2}(0).$$

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Trace preservation.

$Tr\{\nu_t\rho_S(0)\} = 1.$

Two ways to find evolved reduced dynamics.

The unitary evolution and partial trace approach.

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Partial trace of Von Neumann equation approach

$\frac{\partial}{\partial t}\rho_{SE}(t) = -\frac{i}{\hbar}[H_{SE}, \rho_{SE}(t)] \rightarrow \frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}Tr_E\{[H_{SE}, \rho_{SE}(t)]\}$

Reduced Von Neumann equation. Path to Born-Markov master equation.

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  $\frac{\partial}{\partial t}\tau(\rho_{SE})(t) = -\frac{i}{\hbar}[\tau(H_I), \tau(\rho_{SE})(t)]$

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- Equivalently.

$$\frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}Tr_E\{[H_I, \rho_{SE}(0)]\}+$$

$$+\frac{i^2}{\hbar^2}\int_0^t dt_1 Tr_E\{[H_I(t), [H_I(t_1), \rho_{SE}(t_1)]]\}$$

Continued

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The Markov approximation

In order for the reduced Von Neumann equation to be Markovian the integrand must be smooth and sharply peaked in the vicinity of $t \approx t_1$. If this holds than we may trade in $\rho_S(t_1) \otimes \rho_E(0)$ for $\rho_S(t) \otimes \rho_E(0)$.

Born-Markov master equation

$$\frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}Tr_E\{[H_I, \rho_S(0) \otimes \rho_E(0)]\}+$$
$$+\frac{i^2}{\hbar^2}\int_{-\infty}^{t} dt_1 Tr_E\{[H_I(t), [H_I(t_1), \rho_S(t) \otimes \rho_E(0)]]\}$$

Decoherence Models

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- $$\frac{\partial}{\partial t}\rho_S(t) = \frac{-i}{2}(\omega_a + \Delta\omega_a)[\sigma_z, \rho_S(t)] + \gamma D[\sigma_-]\rho_S(t). \qquad (1)$$

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- $D[\sigma_-]\rho := \sigma_-\rho\sigma_+ - \frac{1}{2}(\sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-)$

Solution lives in the Bloch sphere

$\rho_S(t) = \frac{1}{2}[I_2 + x(t)\sigma_x + y(t)\sigma_y + z(t)\sigma_z]$, $Tr\{\rho_S^2(t)\} \leq 1$ therefore

$x^2(t) + y^2(t) + z^2(t) \leq 1$.

- $\frac{\partial}{\partial t} z(t) = Tr\{\sigma_z \frac{\partial}{\partial t} \rho_S(t)\}$
- $\frac{\partial}{\partial t} y(t) = Tr\{\sigma_y \frac{\partial}{\partial t} \rho_S(t)\}$
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Using the *Lindblad* Master equation to substitute for $\frac{\partial}{\partial t} \rho_S(t)$ these equations become

- $\frac{\partial}{\partial t} z(t) = -\gamma(z(t) + 1)$
- $\frac{\partial}{\partial t} y(t) = (\Delta\omega_a) x(t) - \frac{\gamma}{2} y(t)$
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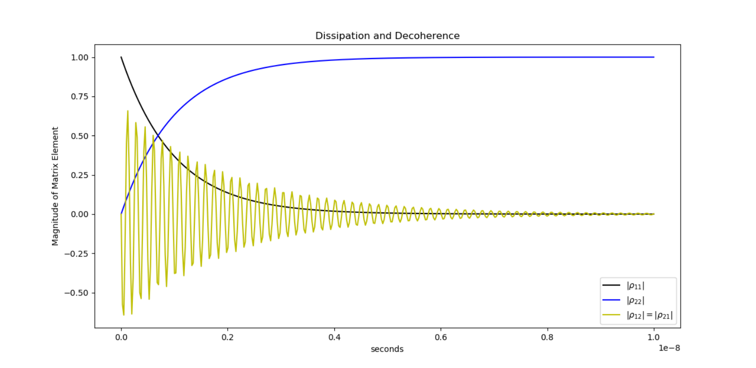
with solutions

- $z(t) = 2e^{-\gamma t} - 1$
- $y(t) = -e^{-\frac{\gamma t}{2}} \sin((\omega_a + \Delta\omega_a)t)$
- $x(t) = e^{-\frac{\gamma t}{2}} \sin((\omega_a + \Delta\omega_a)t)$.

Continued



$$\rho_S(t) \to \begin{bmatrix} e^{-\gamma t} & e^{-\frac{\gamma t}{2}} \sin((\omega_a + \Delta_a)t)\frac{(1+i)}{2} \\ e^{-\frac{\gamma t}{2}} \sin((\omega_a + \Delta_a)t)\frac{(1-i)}{2} & 1 - e^{-\gamma t} \end{bmatrix}$$

General master equation for finite dimensional *Hilbert* space $\mathscr{H}_S$

$$\frac{\partial}{\partial t}\rho_S(t) =$$
$$-\frac{i}{\hbar}[H_S', \rho_S(t)] + \sum_{ij}^{N^2} \alpha_{ij}(t)\{F_i\rho_S(t)F_j^\dagger - \frac{1}{2}F_j^\dagger F_i\rho_S(t) - \frac{1}{2}\rho_S(t)F_j^\dagger F_i\}$$
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Connecting back to dynamical maps

$$\frac{\partial}{\partial t}\rho_S(t) = \frac{\partial}{\partial t}\nu_t\rho_S(0) = \frac{\partial}{\partial t}e^{\mathscr{L}t}\rho_S(0) = \mathscr{L}\nu_t\rho_S(0) = \mathscr{L}\rho_S(t)$$

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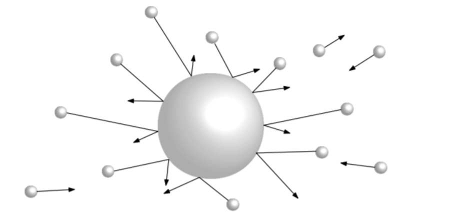
Quantum dynamical semigroup

$\mathscr{L}$ is the generator of the dynamical semigroup $\{\nu_t = e^{\mathscr{L}t}|t \geq 0\}$.

Collisional decoherence, recoilless case.



$$|x\rangle|E\rangle \xrightarrow{t} |x\rangle|E_x\rangle = |x\rangle S_x|E\rangle.$$

S_x is the so called S-matrix, a unitary operator. S-matrix simply maps free particle in-states to free particle out-states and excludes information about the interaction.

Evolution of some state $\phi(x) \in L^2(\mathbb{R})$, $|E\rangle \in \mathscr{H}_E$.

$\{\int dx\phi(x)|x\rangle\}|E\rangle \xrightarrow{t} \int dx\phi(x)|x\rangle S_x|E\rangle$

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Reduced Density matrix

$$\rho_S(x,y) = \phi(x)\phi(y)^* \xrightarrow{t} \rho_S(x,y)\langle E|S_y^\dagger S_x|E\rangle.$$

Evolution of some state $\phi(x) \in L^2(\mathbb{R})$, $|E\rangle \in \mathscr{H}_E$.

$\{\int dx \phi(x)|x\rangle\}|E\rangle \xrightarrow{t} \int dx \phi(x)|x\rangle S_x|E\rangle$

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Scattered photons Long-Wavelength limit

$\langle E|S_y^\dagger S_x|E\rangle \approx e^{-\Lambda t(x-y)^2}$

- $\rho_S(x, x') \xrightarrow{t} \rho_S(x, x') e^{-\Lambda t (x - x')^2}$.

  This implies $\rightarrow \rho_S(x, x', t) = e^{-\Lambda t (x - x')^2} \rho_S(x, x', 0)$.

Collisional Decoherence

- $\rho_S(x, x') \xrightarrow{t} \rho_S(x, x') e^{-\Lambda t (x - x')^2}$.

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- The above is a solution to the differential equation

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  $i\hbar\frac{\partial\rho_S(t)}{\partial t} = -i\Lambda[x,[x,\rho_S(t)]]$. (Master equation)

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- $i\hbar\frac{\partial \rho_S(t)}{\partial t} = [\frac{p^2}{2m}, \rho_S(t)] - i\Lambda[x, [x, \rho_S(t)]]$. (Including intrinsic dynamics, Master equation).

Scattering constant Λ and decoherence timescale $\tau_{\Delta x}$.

Different values of Λ

Environment	**Λ for dust grain, $10^{-3}cm$**	**Λ for dust particle, $10^{-5}cm$**
Cosmic background radiation	10^{6}	10^{-6}
300k photons	10^{19}	10^{12}
Sunlight on earth	10^{21}	10^{17}
Air molecules	10^{36}	10^{32}
Laboratory vacuum	10^{23}	10^{19}

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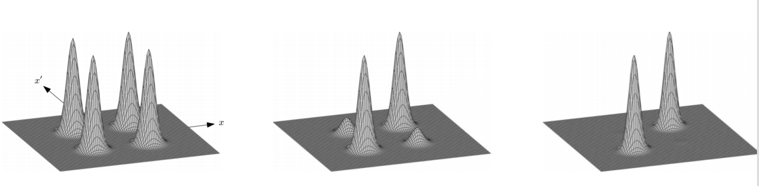
Decoherence timescales, $\tau_{\Delta x} := \frac{1}{\Lambda(\Delta x)^2}$

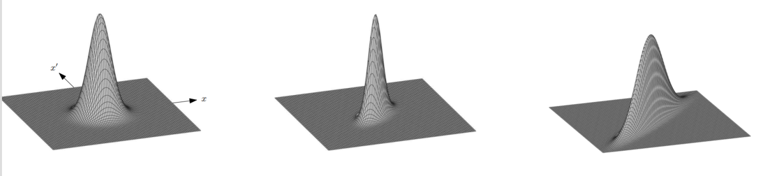
Environment	Λfor Dust grain, $10^{-3}cm$
Cosmic background radiation	1
Photons at room temperature	10^{-18}
Best laboratory vacuum	10^{-14}
Air at normal pressure	10^{-31}

Superposition of two localized Gaussians, just decoherence.



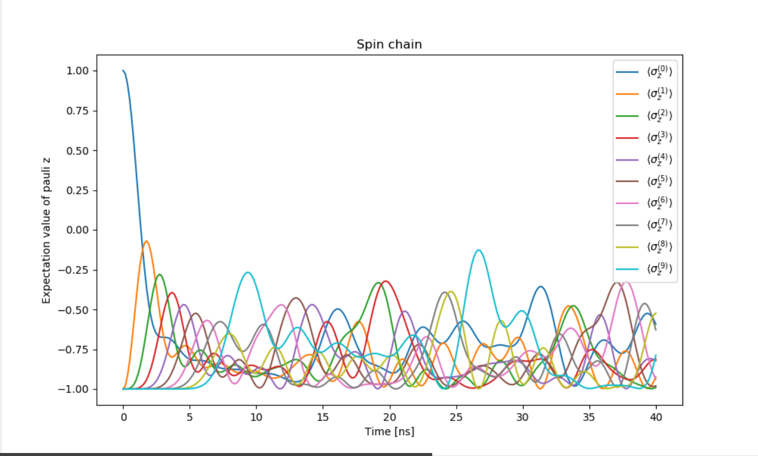
$$\phi(x, t = 0) = N_1 e^{-(x-a_1)^2} + N_2 e^{-(x-a_2)^2}$$

Evolution of a Gaussian initial state, decoherence and delocalization.



The probability distribution of our particles position is $P(x, t) := \rho_S(x, x, t)$.

- $H = -\frac{1}{2}\sum_n^N h_n \sigma_z^n - \frac{1}{2}\sum_n^{N-1}[J_x^n \sigma_x^n \sigma_x^{n+1} + J_y^n \sigma_y^n \sigma_y^{n+1} + J_z^n \sigma_z^n \sigma_z^{n+1}]$
- $H \in \mathscr{B}(\mathbb{C}^{\otimes 2N})$
- Let $N = 10$ and $|\psi(0)\rangle = |1000000000\rangle \in \mathbb{C}^{\otimes 2N}$



Spin chains decoherence

- $H = -\frac{1}{2}\sum_n^N h_n \sigma_z^n - \frac{1}{2}\sum_n^{N-1}[J_x^n \sigma_x^n \sigma_x^{n+1} + J_y^n \sigma_y^n \sigma_y^{n+1} + J_z^n \sigma_z^n \sigma_z^{n+1}]$
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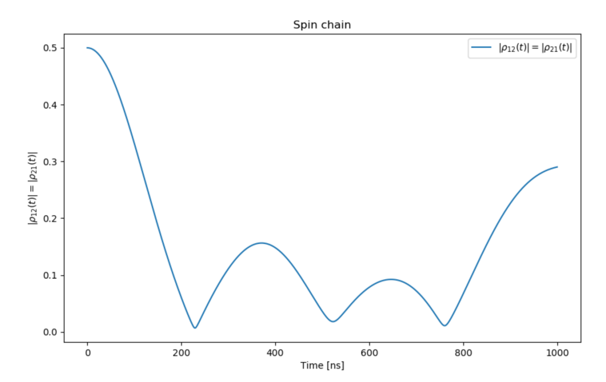


Decoherence for spin environments in the large N limit.

Decoherence terms damping

For large N, the decoherence terms follow an approximate Gaussian dependence $e^{-\Gamma^2 t^2}$. Where Γ depends on environmental properties and coupling constants J_i^n.

Quantum Brownian Motion

Hamiltonian

- $H_E = \sum_i (\frac{1}{2m_i} p_i^2 + \frac{1}{2} m_i \omega_i^2 q_i^2)$.
- $H_I = x \otimes \sum_i c_i q_i$.
- $H_S = \frac{1}{2M} p^2 + \frac{1}{2} M \Omega^2 x^2$.

Master equation under Born-Markov approximation

$$\frac{\partial}{\partial t}\rho_S(t) = -\frac{i}{\hbar}[H_S + \frac{1}{2}M\Delta^2 x^2, \rho_S(t)] - \frac{i\gamma}{\hbar}[x, \{p, \rho_S(t)\}] - D[x[x, \rho_S(t)]] - \frac{f}{\hbar}[x, [p, \rho_S(t)]]$$

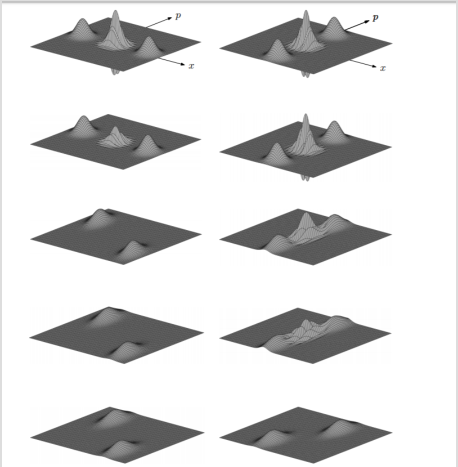
Uncertainty of x

$$\Delta X^2(t) = \frac{\hbar^2 D}{2m^2\gamma^2} t.$$

Wigner transform

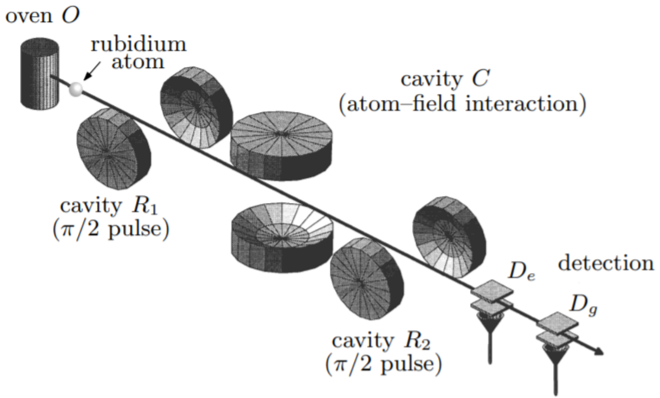
$W(x, p) := \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} e^{i\frac{ipy}{\hbar}} \rho(x + \frac{y}{2}, x - \frac{y}{2}).$

Monitoring Coherences with Wigner transform



Decoherence in the lab

Photons states in cavity



Schematic steps

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- $\frac{1}{2}(|\alpha e^{-i\xi}\rangle + |\alpha e^{i\xi}\rangle)|g\rangle + \frac{1}{2}(-|\alpha e^{-i\xi}\rangle + |\alpha e^{i\xi}\rangle)|e\rangle \xrightarrow{Detection}$
- $|\pm\rangle = \frac{1}{\sqrt{2}}(|\alpha e^{i\xi}\rangle \pm |\alpha e^{-i\xi}\rangle)$

Photon field

State of photonic field left behind in cavity.
$|\pm\rangle = \frac{1}{\sqrt{2}}(|\alpha e^{i\xi}\rangle \pm |\alpha e^{-i\xi}\rangle)$ If the atom is detected to be in the

state $|g\rangle$ the field is in the field $|+\rangle$, if the atom is detected in the state $|e\rangle$ the field is in the state $|-\rangle$.

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Mesoscopic distinguishability.

To measure the degree to which components $|\alpha e^{i\xi}\rangle$ and $|\alpha e^{-i\xi}\rangle$ represent mesoscopically or macroscopically distinguishable states- we consider $|\langle\alpha e^{i\xi}|\alpha e^{-i\xi}\rangle|^2 = e^{-4|\alpha|^2 \sin^2 \xi}$.

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$$|\langle e^{i\xi}|\alpha e^{-i\xi}\rangle|^2 < 3x10^{-5}.$$

Decoherence

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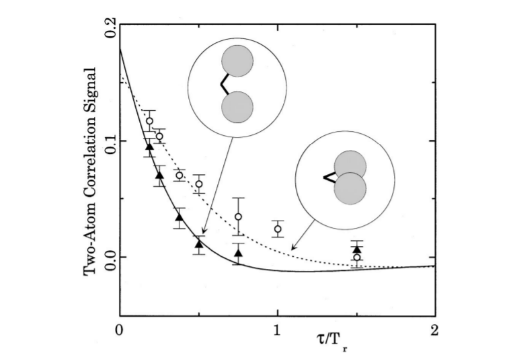
- Decoherence is expected.
- To measure the time dependence of decoherence a second rubidium atom is sent through at varying wait time.
- It can be shown that under zero decoherence $P_{ee} = 1$, probability of first and second atom being detected in the excited state. On the other hand, under full decoherence $P_{eg} = 1$, the probability of finding the second atom in the ground state is 1.
- A useful measuring tool of decoherence.

$$\eta(\tau) = P_{ee}(\tau) - P_{eg}(\tau).$$

Two-Atom Correlation Signal. $\eta(\tau) := P_{ee}(\tau) - P_{eg}(\tau)$.



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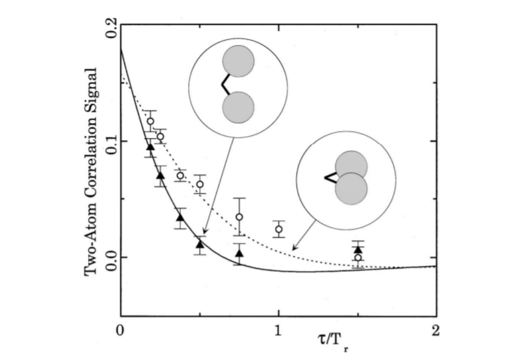

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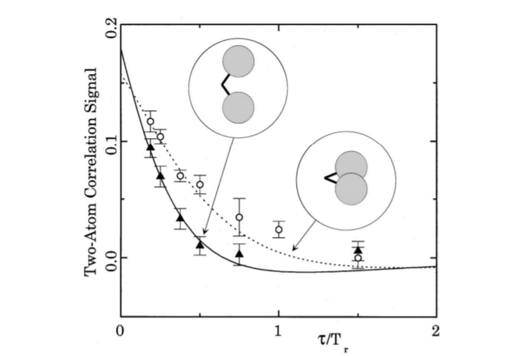

Decoherence time scale $T_d = \frac{T_r}{2|\alpha|^2 \sin^2 \xi}$. T_r damping time of cavity.

Decoherence free subspaces

From earlier

- Total system has some *Hilbert* space $\mathscr{H}_S \otimes \mathscr{H}_E$

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- $|\psi_{\mathsf{S}}\rangle \otimes |\psi_{\mathsf{E}}\rangle := |\psi_{\mathsf{SE}}\rangle \in \mathscr{H}_{\mathsf{S}} \otimes \mathscr{H}_{\mathsf{E}} := \mathscr{H}_{\mathsf{tot}}$.

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- $|\psi_S\rangle \otimes |\psi_E\rangle := |\psi_{SE}\rangle \in \mathscr{H}_S \otimes \mathscr{H}_E := \mathscr{H}_{tot}$.
- Dynamics provided by Schrödinger's equation.
  $i\hbar\partial_t|\psi_{SE}(t)\rangle = H|\psi_{SE}(t)\rangle$ where $H = H_S + H_E + H_I$, a *Hermitian* operator in $\mathscr{B}(\mathscr{H}_{SE})$.
- $|\psi_{SE}(t)\rangle = e^{-\frac{it}{\hbar}H}$. Just like before.

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- $\rho_S(t) = Tr_E\{e^{\frac{-it}{\hbar}\sum_k S_k \otimes E_k}(\{\sum_{1,m} c_m c_m^* |s_l\rangle\langle s_m|\} \otimes |E_0\rangle\langle E_0|)e^{\frac{it}{\hbar}\sum_k S_k \otimes E_k}\}$

This partial trace in general reduces to some state of the form,

$$\rho_S(t) = \sum_{l,m} a_l(t) a_m^*(t) |s_l\rangle\langle s_m|$$

with $a_l(t)a_m^*(t) \to 0$ as $t \to \infty$ for$l \neq m$.

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$$|\psi_{SE}(t)\rangle = e^{\frac{-it}{\hbar}\sum_k S_k\otimes E_k}\sum_l c_l|s_l\rangle \otimes |E_0\rangle =$$

$$= \sum_l c_l e^{\frac{-it}{\hbar}\sum_k \lambda_k I_A\otimes E_k}|s_l\rangle \otimes |E_0\rangle = \sum_l c_l|s_l\rangle \otimes [e^{\frac{-it}{\hbar}\sum_k \lambda_k E_k}|E_0\rangle]$$

Partial Trace

Let us now partial trace the corresponding density matrix.

$$\rho_S(t) = \sum_{l,m} c_l c_m^* |s_l\rangle\langle s_m| Tr_E[e^{\frac{-it}{\hbar}\sum_k \lambda_k E_k}|E_0\rangle\langle E_0|e^{\frac{it}{\hbar}\sum_k \lambda_k E_k}]$$

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The trace term is just one since density matricese have trace one under unitary evolution.

$$\rho_S(t) = \sum_{l,m} c_l c_m^*$$

.

Decoherence free!!

Example, symmetric dephasing

Consider a system of N qubits coupled to its environment in the follwoing way.

$$|0\rangle_j \to |0\rangle_j$$

$$|1\rangle_j \to e^{i\phi}|1\rangle_j.$$

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The dephasing process evolves our system into the following

state.

$$|\psi\rangle_\phi = \bigotimes_{j=1}^{N}(a_j|0\rangle_j + b_j e^{i\phi}|1\rangle_j)$$

with a probability p_ϕ

The ensemble $\{|\psi\rangle_\phi, p_\phi\}$ can be expressed equivalently as a mixed state.

$$\rho = \int p_\phi |\psi\rangle_\phi \langle\psi| d\phi$$

Example continued

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$$|\psi\rangle_\phi\langle\psi| \to \bigotimes_{j=1}^{N} \begin{bmatrix} |a_j|^2 & a_j b_j^* e^{-i\phi} \\ a_j^* b_j e^{i\phi} & |b|^2 \end{bmatrix}.$$

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For a Gaussian distribution $p_\phi = (4\pi\alpha^{-\frac{1}{2}})e^{\frac{-\phi^2}{4\alpha}}$ we have

$$\rho \rightarrow \bigotimes_{i=1}^{N} \begin{bmatrix} |a_j|^2 & a_j b_j^* e^{-\alpha} \\ a_j^* b_j e^{-\alpha} & |b|^2 \end{bmatrix}.$$

There is indeed decoherence present, lets look for some DFS.

Example continued

For starters lets consider the case $N = 2$. The dephasing for each of the constituents of the corresponding *Hilbert* space $\mathbb{C}^2 \otimes \mathbb{C}^2$ is summarized by the following.

- $|00\rangle \to |00\rangle$
- $|01\rangle \to e^{i\phi}|01\rangle$
- $|10\rangle \to e^{i\phi}|10\rangle$
- $|11\rangle \to e^{2i\phi}|11\rangle$.

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$$Span\{|01\rangle, |10\rangle\}?$$

check...

$$|\psi\rangle = a|01\rangle + b|10\rangle \to ae^{i\phi}|01\rangle + be^{i\phi}|10\rangle = e^{i\phi}|\psi\rangle$$

It works!!

Example continued

For $N = 3$ the largest DFS is $Span\{|001\rangle, |010\rangle, |100\rangle\}$ or $Span\{|011\rangle, |101\rangle, |110\rangle\}$

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$$\frac{|max[Dim(DFS)] - 2^N|}{2^N} \to 1.$$

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textbook application of stirling's formula yields the following.

$$\frac{|max[Dim(DFS)] - 2^N|}{2^N} \to 1.$$

The dimension of the optimal DFS becomes relatively close to the dimension of the system for large N.

Future work

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- Study robustness of DFS under perturbations.

Future work

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- Find a way to simulate large spin environments.

- Study robustness of DFS under perturbations.
- Find a way to simulate large spin environments.
- Decoherence theory in infinite dimensional Hilbert spaces and extending SBS theory to such systems.

Thanks

Thank you for your time.

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